Communication for maths

On the formal presentation of differentiation – part 2

On writing mathematics

- Maths is not just notation. It is
  - turns of phrases and forms of writing;
  - logic and organisation in the writing;

  (see example styles 1, 2 and 3 below)

- Mathematics proceeds step-by-step, and uses clear argumentation (logic, algebra, etc.)

On writing mathematics

Style 1

1) Let f be a continuous function on a closed interval $[a, b]$ and differentiable on the open interval (a, b). If $f^{'}(x) > 0$ for all $x \in (a, b)$ then f is increasing on $[a, b]$.

On writing mathematics

Style 2

2) Consider the interval $I = [a, b]$, and let the continuous function f be defined on I. Given that f is differentiable on the open interval (a, b), $f'(x) > 0$ for all $x \in (a, b)$ implies f is increasing on $[a, b]$.

On writing mathematics

Style 3

3) We say that a continuous function f is increasing on a closed interval $I = [a, b]$, when $f'(x) > 0$ for all $x \in (a, b)$, provided that f is differentiable on the open interval (a, b).

On writing mathematics

Logical presentation of steps

Let $x_1, x_2 \in [a, b]$ such that $x_2 > x_1$.

Then $f'(x_1) > 0 \Rightarrow \lim_{x_2 \to x_1} \left(\frac{f(x_2) - f(x_1)}{x_2 - x_1} \right) > 0$

On writing mathematics

Logical presentation of steps

Therefore $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} > 0$, implying

$$f(x_2) - f(x_1) > 0.$$

On writing mathematics

<u>*Logical presentation of steps*</u>

Hence $f(x_2) > f(x_1) \Rightarrow f$ is increasing on $[a, b]$

for all $x_1, x_2 \in [a, b]$ $\blacksquare$

Some terminology and phrasing

1) "Find the stationary points of …" means

____

2) "Classify the stationary points of …" means

____

Some terminology and phrasing

3) "Find and classify the stationary points of …"

means

____

Some terminology and phrasing

4) “Find maximum and minimum points of …”

means

____

Finding and classifying stationary points

Examples

Consider the following question:

Find the stationary points of

$$\frac{dy}{dx} = x^3 + \frac{x^2}{2} - 2x - 2$$

Finding and classifying stationary points

Example

What is missing from the following?

$$y = \frac{1}{3}x^3 + x^2 + 3x$$

$$y' = x^2 + 2x + 3$$

$$\therefore \quad x^2 + 2x + 3 = 0$$

Finding and classifying stationary points

Example

There was no justification as to why we are putting the quadratic equal to zero:

$$y = \frac{1}{3}x^3 + x^2 + 3x$$

$$y' = x^2 + 2x + 3$$

FOR S.P. $y' = 0$

$$\therefore x^2 + 2x + 3 = 0$$

Finding and classifying stationary points

Example

What is wrong with the following solution?

Solution: $3x^2 + x - 2$

$f'(-1) = 0$, $(x+1)$ is factor

$(x+1)(3x-2) = 0$

-1 $+\frac{1}{2}$ t.p.

$x = -1, +\frac{2}{3}$

$\Rightarrow$ max at $x = -1$ & min at $x = +\frac{2}{3}$

$y = -\frac{1}{2}$

Summary about finding and classifying stationary points

- The following are some (not all) general points in the presentation of maths, specifically relating to the above examples:
  - No free-standing expressions;
  - Justify step;
  - No scratch marks or rough work;
  - Clean presentation of writing;

Summary about finding and classifying stationary points

- The following are key points in the presentation of finding and classifying stationary points:
  - Present the derivative function separately from the fact that it equals zero, i.e. separate the step involving the equation "y′ = …" from the equation "…= 0"
  - Say something like, "… For stationary points, y′(x) = 0, hence …"

Summary about finding and classifying stationary points

- The following are key points in the presentation of finding and classifying stationary points:
  - Test for local extrema, and **show** the test you use; Also test for global extrema where relevant;
  - State the value of the extreme points and/or the coordinates of the extreme points as necessary.

All the usual rules of presentation apply

- Reminder of some key points of presentation:
  - Clean and clear presentation of solutions;
  - Use "$\therefore$" or ""hence", or "therefore", or "$\Longrightarrow$", … as appropriate;
  - Justifying steps where required;
  - No free-standing expressions;

All the usual rules of presentation apply

- The most important things in the presentation of maths for the exam are:
  - Clean and clear presentation of solutions;
  - Use "∴" or ""Hence", or "As such", etc.;
  - Justifying steps where required;
  - No free-standing expressions;
  - Give complete answers at each intermediate stage;
  - Aligning the "=" symbol;
  - Exact vs approximate values.

Examples

Exercise 1 – Finding and classifying stationary points: Find the errors in the maths presentation of the problem handed out.

Example 1: An optimisation problem: Find the errors in the maths presentation of the problem handed out.

Summary about optimisation

- The following are key points in the presentation of optimisation problems:
  - State the domain for the function and the domain of its derivative;
  - Test for extrema: Test $dy/dx = 0$, and test end points;
  - Answer the question. State the value of the maximum volume or minimum surface area, or minimum time, etc.

About the term 1 exams

- The most important things in the presentation of maths for the exam are:
  - Clean and clear presentation of solutions;
  - Use "$\therefore$" or ""Hence", or "As such", etc.;
  - Justifying steps where required;
  - No free-standing expressions;
  - Give complete answers at each intermediate stage;
  - Aligning the "=" symbol;
  - Exact vs approximate values.

About the term 1 exams

Exam booklets

- You will be given 3 answer booklets
  - One booklet will be for solutions to section A questions. Please answer only sec A questions here;
  - One booklet will be for solutions to section B, question 1 and question 2. Please answer at most two of these sec B questions here;
  - One booklet will be for solutions to section B, question 3 and question 4. Please answer at most two of these sec B questions here.

======================

Appendix

======================

No free-standing expressions

Examples

What is wrong with the following solution?

Solution

$3x^2 + x - 2$

Since $f(-1) = 0$, $(x - 1)$ is a factor

and $(x+1)(3x+2) = 0$

Hence $x = -1, -\frac{2}{3}$

No free-standing expressions

Examples

The first line is a “free-standing” expression that does not refer to anything.

What is the first step the answer to?

Solution

$3x^2 + x - 2$

Since $f(-1) = 0$, $(x - 1)$ is a factor

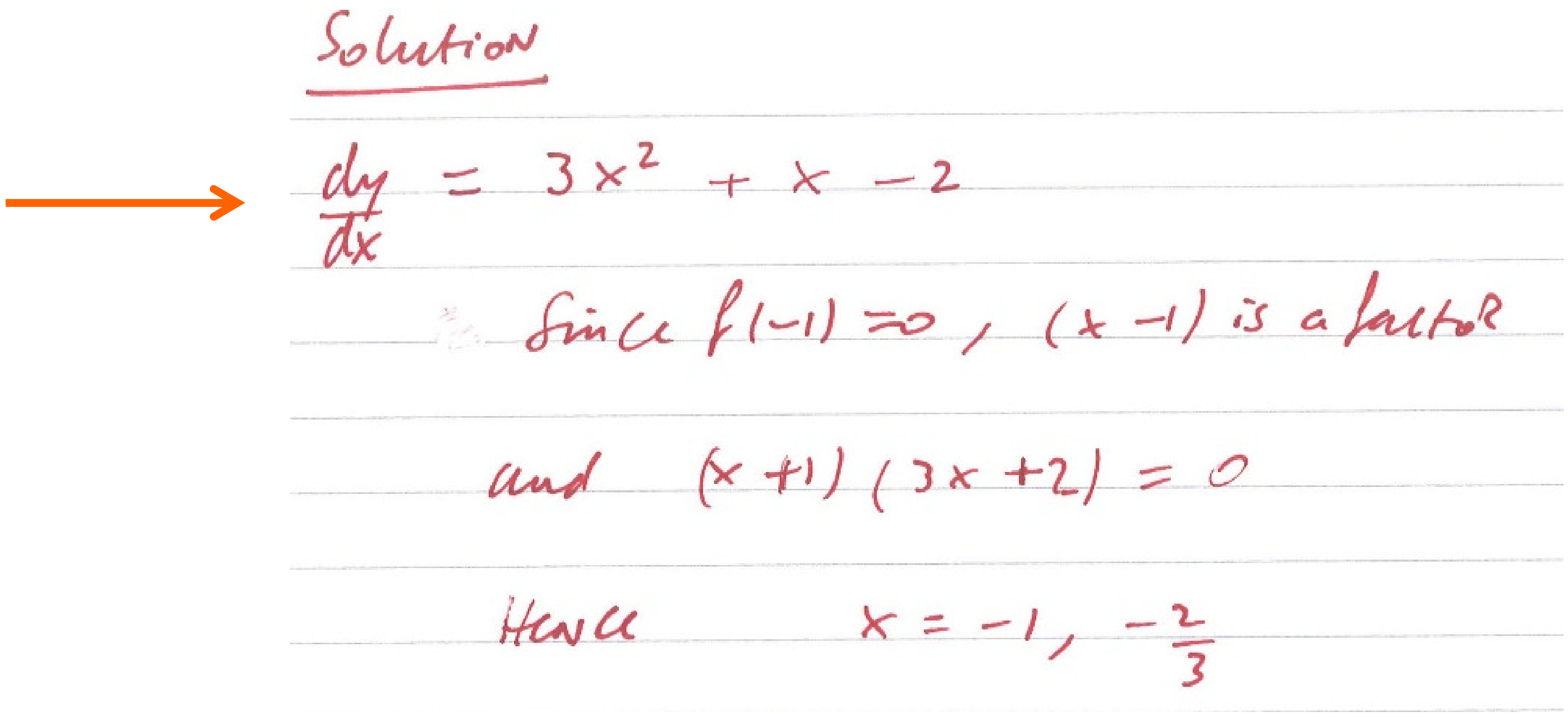
and $(x + 1)(3x + 2) = 0$

Hence $x = -1, -\frac{2}{3}$

No free-standing expressions

Examples

The first line is a “free-standing” expression that does not refer to anything.



Give complete answers at intermediate stages;

Example

Examples and exercises

Exercise 1 – Finding and classifying stationary points: Find the errors in the maths presentation of the problem handed out.

Example 2: An optimisation problem
(*volume of a box from the cut-out corners of a rectangle*)